First-order transition in a particle deposition-evaporation model

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We introduce a simple stochastic growth model where particles of two different species are deposited and evaporated. In the model, a randomly chosen particle of two species is deposited at a rate p and a particle on the edge of the plateau of the interface is evaporated at a rate $1-p$. When $p<p_{c1}=0.4985(2)$ and $p>p_{c2}=0.5015(5)$, the velocity of the interface is zero. When $p_{c1}\leqslant p\leqslant p_{c2}$, however, the interface grows with a constant velocity. At both p_{c1} and p_{c2}, the velocity of the interface changes from zero to a constant value discontinuously. The first-order transitions in our model are related to a nonequilibrium phase transition from an active to an inactive phase at the bottom layer of the interface. Interestingly, the first-order transition at p_{c1} is triggered by the combination of the parity conserving and the directed percolation dynamics. We explain why the transitions in our model are of first order. Moreover, our model shows two nonequilibrium roughening transitions at p_{c1} as well as at $p_r[=0.444(2)]$.

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For last several years there have been many studies about the nonequilibrium binding-unbinding (BU) transition of a growing interface in a $1+1$ dimensional system, via stochastic growth models [1–4]. These models show a nonequilibrium BU transition from a phase where the interface is bound to the wall to a moving phase where the interface becomes unbound. At the BU transition, the velocity of the interface changes from zero to a nonzero value. Often the BU transition is accompanied by a nonequilibrium roughening (NR) transition. The NR transition in $1+1$ dimensions is an interesting phenomenon because the interface under thermal equilibrium in $1+1$ dimensions is always rough and thus does not exhibit a roughening transition. In higher dimensions, however, the interface under thermal equilibrium can undergo a roughening transition at a certain critical temperature.

The BU or NR transitions studied in Refs. [1–4] are known to be related to phase transitions into absorbing states, which belong to the directed percolation (DP) [5–7] or the parity conserving (PC) universality class [8–12]. For example, in the growth model introduced by Alon, Evans, Hinrichsen, and Mukamel (the AEHM model) [1], some features of the BU transition are known to be related to DP. In the dimer deposition-evaporation model [2], however, some features of the BU transition are known to be related to PC. In above two models, an absorbing transition in the DP or the PC class emerges at a particular reference height of the interface, i.e., at the bottom layer of the interface (the level of minimum height). More specifically, the sites touching the bottom layer correspond to the active sites of DP or PC. Therefore, in the active phase of DP or PC, the interface fluctuates close to the reference height so that the interface is smooth and bound at the bottom layer. On the other hand, in the inactive phase of DP or PC, the interface detaches from the reference height and evolves into a rough state. A common feature of all models showing the BU or NR transition is the change of the velocity of the interface depending on the deposition rate of a particle. The velocity of the interface is zero in the active phase of DP or PC, but it is nonzero in the inactive phase. In most of the models, the BU transition is a *continuous* one.

Recently, there have been a few studies about a first-order BU transition [13,14]. In those studies, a hard-core wall at zero height is introduced and a first-order BU transition takes place because of the binding force between the interface and the wall. To our knowledge, in all models where the growing interface does not interact with the wall, BU transitions were continuous transitions. However, as we are going to show in the present paper, the first-order BU transition can occur even in the case where there is no interaction between the interface and the wall. To this end, we introduce a simple growth model exhibiting a first-order BU transition. In our model, there is no interaction with a hard-core wall at zero height. Rather, it turns out that the first-order BU transition in our model originates from the combination of the PC and DP dynamics.

Our stochastic model is defined as follows. Initially there is no particle in the system, i.e., the interface is flat. Each time, either deposition or evaporation of a particle occurs at a randomly selected site. Then deposition and evaporation takes place at rates p and $1-p$, respectively. In the deposition process, an A or a B particle is deposited with equal probability. The interface height at a site i is represented by an integer height variable h_i. In our model, three restrictions are imposed. One is the restricted solid-on-solid condition $|h_i-h_{i+1}|\leqslant 1$, which is imposed at all sites in the deposition and evaporation processes. Another restriction is that a particle can evaporate only at the edges of plateaus of the interface. Finally, we assume an infinitely strong repulsive interaction between two nearest-neighbor particles of different kind in the deposition process. For example, the deposition of an A particle at a randomly selected site is not allowed if there is at least one B particle among the three nearest-neighbor sites of the deposited particle after deposition (see Fig. 1). The dynamic rule of an A particle in our model is symmetric with respect to that of a B particle and vice versa.

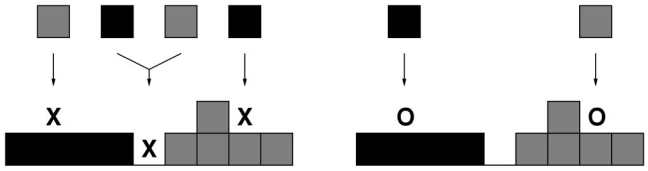

FIG. 1. Schematic representation of the stochastic growth rule of the model. A black particle (A particle) cannot be deposited at a randomly selected site if the particle touches a gray particle (B particle) after deposition. A gray particle also cannot be deposited at a randomly selected site if the particle touches a black particle after deposition.

Without the repulsive interaction, the growth rule of our model becomes the same as that of the AEHM model.

If the deposition rate of a particle, p, is very small, only few deposited particles can stay on the substrate for a short lifetime before evaporation. Therefore, the interface does not grow, i.e., it is bound to the substrate. As p increases, more particles stay on the substrate and form large islands. When p exceeds the critical value p_{c1}, the interface detaches from the substrate and grows with a constant velocity. However, because of the interaction the interface can only detach if the bottom layer is completely covered by particles of a single species such as $AAA\cdots AAA$ or $BBB\cdots BBB$. After all the sites of the bottom layer are filled completely with one kind of particles, deposition as well as evaporation of particles does not occur anymore at the bottom layer. In that case, two symmetric states at the bottom layer, $AAA\cdots AAA$ or $BBB\cdots BBB$, can be interpreted as Z_2-symmetric absorbing states.

If deposition and evaporation occur only at the bottom layer, the dynamic rules in our model would resemble those of a model with Z_2-symmetric absorbing states, which is known to belong to the PC class [12]. However, after the bottom layer has been completely filled with A particles (B particles), only A particles (B particles) can be deposited because of the interaction between nearest-neighbor particles. In that case, the growth rule in our model becomes the same as that of the AEHM model showing DP dynamics at a particular reference height. From now, we will denote the AEHM model type of growth process as DP-type growth and the growth process related to the PC dynamics at the reference height as PC-type growth. In our model, the dynamics of the growing interface is affected by the PC-type growth at the bottom layer, but it is affected by the DP-type growth at all following layers.

We carried out Monte Carlo simulations for our model. We measured the velocity of the interface by changing the deposition rate p from 0 to 1. For small p, the velocity of the interface is zero. The velocity maintains zero until $p<p_{c1}=0.4985(2)$. However, the velocity is nonzero for $p_{c1}\leqslant p\leqslant p_{c2}=0.5015(5)$ and again becomes zero for $p>p_{c2}$. Here the velocity of the interface is defined as $v(t)=(1/L)\Sigma_{i=1}^{L}[h_i(t)-h_i(t-1)]$, where L denotes the system size. A surprising feature is that the velocity suddenly jumps from zero to a nonzero constant value, exhibiting a first-order transition at p_{c1} (see Fig. 2). In order to check whether this unexpected transition is really of first order, we measured the velocity of the interface by continuously decreasing p from 0.5 after the velocity saturates to a constant value at $p=0.5$. We found that the velocity decreases *continuously* from 0.08 at $p=0.5$ to 0 at $p=0.444(2)$. Therefore, the velocity exhibits a history dependent behavior, confirming that the BU transition is of first order. The first-order BU transition originates from the fact that the dynamics of the growing interface is different at the bottom and the other layers. At the BU transition point p_{c1}, the growth of the interface is affected by the PC-type growth at the bottom layer and the suppression effect of the heaped particles on the bottom layer, where the heaped particles are made by the DP-type growth process. These two effects hinder the interface from growing. However, as soon as the bottom layer is filled completely with one kind of particles, the velocity of the interface is governed by the DP-type growth process in the same way as in the AEHM model [1]. The deposition rate of a particle at p_{c1} is large enough for the interface to grow with a large velocity if the growth in our model takes place only by the DP-type growth process. We found that, from the computer simulations starting from the initial condition completely filled with A particles below bottom layer, the velocity of the interface is nonzero for $p>p_r\,[=0.444(2)]$. Moreover, the velocity follows the upper velocity curve in Fig. 2 for $p>p_r$ and increases continuously until $p=1$. This fact again confirms that our model shows a first-order phase transition at p_{c1}. Therefore, the coexistence of the DP- and PC-type growth processes generates the first-order BU transition in our model.

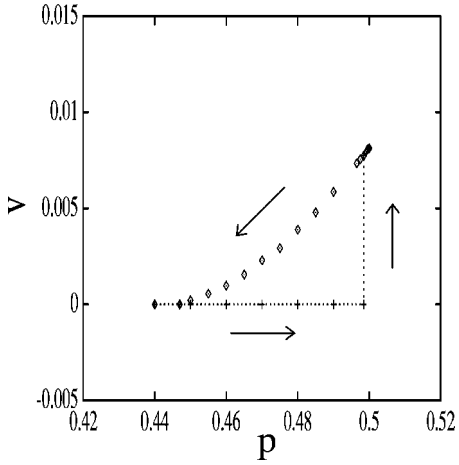

FIG. 2. Plot of the velocity of the interface, v, vs deposition rate of a particle, p, for the system size $L=1024$. When p increases from 0 to 0.5, the velocity of the interface jumps from 0 to 0.007 638 discontinuously at p_{c1} ($=0.4985$). However, when p decreases from 0.5 to 0.44, the velocity decreases continuously from 0.008 at $p=0.5$ to 0 at $p=0.444$, showing a history dependent behavior.

In order to support this interpretation, we measured the vacant site density $\rho(p,t)$ at the bottom layer. $\rho(p,t)$ has a finite value for $p<p_{c1}$ and $p>p_{c2}$, but it decreases to zero exponentially for $p_{c1}<p<p_{c2}$ as the time increases (see Figs. 3 and 4). At both p_{c1} and p_{c2}, $\rho(p,t)$ decays algebraically as time increases. At p_{c1}, $\rho(p_{c1},t)$ scales as

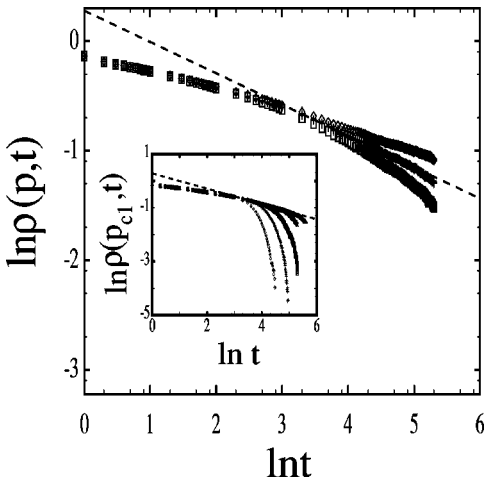

FIG. 3. Plot of $\rho(p,t)$ vs t in double logarithmic scales for the deposition rate $p=0.4975$ (top), $0.4985(=p_{c1})$, and 0.4965 (bottom). The data were obtained for the system size $L=1024$. The line obtained from the least squares fit has the slope $\beta/\nu_{\parallel}=0.285(2)$. Inset: Plot of $\rho(p_{c1},t)$ vs t in double logarithmic scales for the system sizes $L=64$, 128, 256, 512, and 1024 at the critical point p_{c1}.

$$\rho(p_{c1},t)\sim t^{-\beta/\nu_{\parallel}}. \tag{1}$$

From Monte Carlo simulations for different system sizes $L=64$–1024, we measured $\delta_{p_{c1}}(=\beta/\nu_{\parallel})=0.285(2)$, which is in excellent agreement with the value expected in the PC class, 0.286(2) [7]. We also measured the vacant site density $\rho(p,t)$ at the bottom layer at p_{c2}. From Monte Carlo simulations for different system sizes $L=512$–4096, we measured $\delta_{p_{c2}}=0.50(1)$ (see Fig. 4). From these results, we can conclude that the interface in our model grows with a constant velocity only in a very narrow region, $p_{c1}\leqslant p\leqslant p_{c2}$. From the study of the density $\rho(p,t)$, we know that the velocity of the growth interface should be zero in our model for

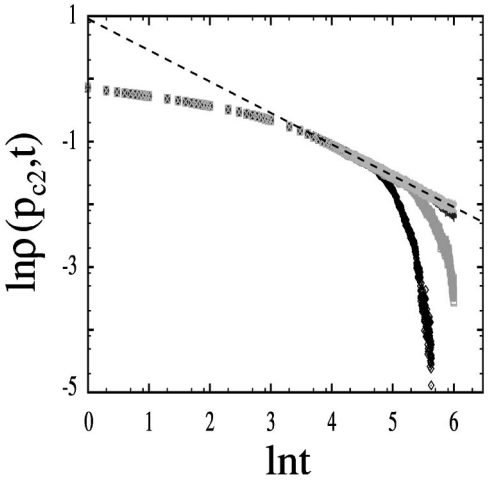

FIG. 4. Plot of $\rho(p,t)$ vs t in double logarithmic scales for the system sizes $L=512$, 1024, 2048, and 4096 at the critical point p_{c2} $(=0.5015)$. The line obtained from the least squares fit has the slope $\beta/\nu_{\parallel}=0.50(1)$.

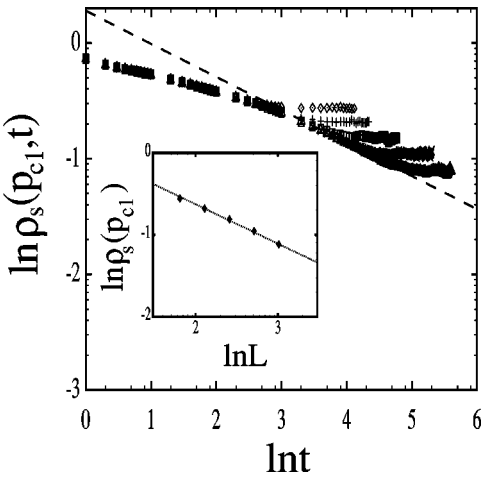

FIG. 5. Plot of $\rho_s(p_{c1},t)$ vs t in double logarithmic scales for the system sizes $L=64$, 128, 256, 512, and 1024. The slope of the dotted line is $\beta/\nu_{\parallel}=0.285$. Inset: Plot of $\rho_s(p_{c1})$ vs L in double logarithmic scales for the system sizes $L=64$, 128, 256, 512, and 1024. The line obtained from the least squares fit has the slope $\beta/\nu_{\perp}=0.48(2)$.

both $p<p_{c1}$ and $p>p_{c2}$. We found that the velocity of the interface is nonzero at p_{c2} for a system with finite size. However, we found that the velocity decreases gradually as the system size increases. We believe that the velocity will become zero in the limit $L\to\infty$ at p_{c2}. Therefore, the BU transition at p_{c2} must be also of first order.

We considered the stationary density $\rho_s(p_{c1},t)$ that is averaged over samples with at least one vacant site at the bottom layer. The density decays as in Eq. (1) before the saturation time $\tau(t<\tau)$ and has a finite value for $t>\tau$. The stationary value of $\rho_s(p_{c1})$ depends on the system size L as $\rho_s(p_{c1})\sim L^{-\beta/\nu_{\perp}}$. We obtained $\beta/\nu_{\perp}=0.48(2)$, which is in relatively good agreement with the expected value from the PC class (see Fig. 5), 0.5 [7].

Next, we considered the interface fluctuation width at p_{c1}, which is defined by $W(L,t)=\langle L^{-d}\Sigma_i[h_i(t)-\overline{h}(t)]^2\rangle^{1/2}$. The width $W(L,t)$ scales as

$$W(L,t)\sim\begin{cases} t^{\zeta/z} & \text{if } t\ll L^z, \\ L^{\zeta} & \text{if } t\gg L^z. \end{cases} \tag{2}$$

Here $\overline{h}$ and d denote the mean height and the substrate dimension. ζ and z are called the roughness and the dynamic exponents. At p_{c1}, the roughness exponent is measured as $\zeta=0.43(2)$ [see Fig. 6(a)]. The interface in our model is smooth for $p<p_{c1}$, i.e., $\zeta=0$, but it is rough at p_{c1}. The value of the roughness exponent jumps from 0 to 0.43 discontinuously at p_{c1}, showing a roughening transition. This result differs from the one obtained from well-known growth models, which show PC- or DP-type dynamics at the reference height [1,3,4]. In all those models, the roughness exponent exhibits a marginal behavior, $\zeta=0$, at the critical point p_c. The value of the roughness exponent increases continuously as p increases from p_c. Therefore, those models ex-

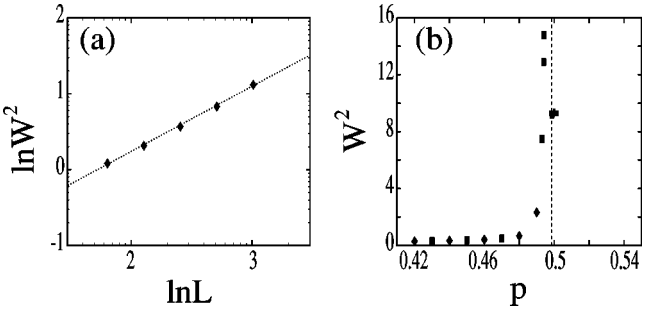

FIG. 6. Plot of the saturated width W^2 vs the system size L at p_{c1} in double logarithmic scales for the system sizes $L=64, 128, 256, 512$, and 1024 (a) and the saturated width W^2 vs p for the system size $L=700$ (b). In Fig. 6(a), the slope of the dotted line is $\zeta=0.43(2)$. In Fig. 6(b), the saturated width increases with p and the value of the width becomes larger near p_{c1} ($=0.4985$) than at p_{c1}. This anomalous behavior originates from the finite size effect. If system size is very large, the width of the interface near p_{c1} will become smaller than at p_{c1}.

hibit the NR transition from a smooth phase with $\zeta=0$ at p_c to a rough one with $\zeta>0$ for $p>p_c$.

We found via computer simulations that the width of the interface in our model seems to be larger for $p<p_{c1}$, where p is near p_{c1}, than at p_{c1} in the case of small system size [see Fig. 6(b)]. But this is a finite size effect. The width of the interface will always become smaller and the value of the roughness exponent will always become 0 for $p<p_{c1}$ if the system size is very large.

We measured the values of the growth and the roughness exponents at $p_r=0.444(2)$ by carrying out computer simulations starting from the initial condition completely filled with A particles below the bottom layer. The roughness exponent can be also obtained from the height-height correlation function $C(x)=\langle(h_{i+x}-h_i)^2\rangle^{1/2}\sim x^{\zeta}$, which should be measured after the interface width reaches a steady state. We found that the interface width increases as $W(t)\sim(\ln t)^{\beta'}$ until it reaches a constant value. We also found that the height-height correlation function follows $C(x)\sim(\ln x)^{\zeta'}$. Then β' and ζ' are about 0.47 and 0.45 at p_r (see Fig. 7). These logarithmic behaviors suggest that $\zeta=0$. We also found that ζ increases continuously as p does from p_r to p_{c1} and becomes 0.43(2) at p_{c1}. Therefore, there occurs the NR (smoothing) transition at $p_r=0.444(2)$ in our model if we decrease p from 0.5 to 0 continuously after the width of the interface reaches a constant value.

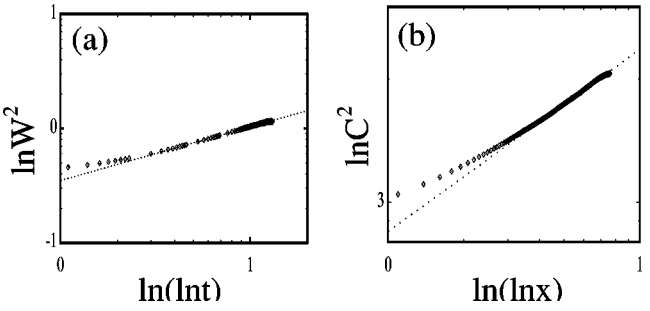

FIG. 7. Plot of the width W^2 vs t (a) and the height-height fluctuation width C^2 vs x (b) in double logarithmic scales for the system size $L=4096$. The data were obtained at p_r. In Fig. 7(a), the slope of the dotted line is $\beta'=0.47$. In Fig. 7(b), the slope of the dotted line is $\zeta'=0.45$.

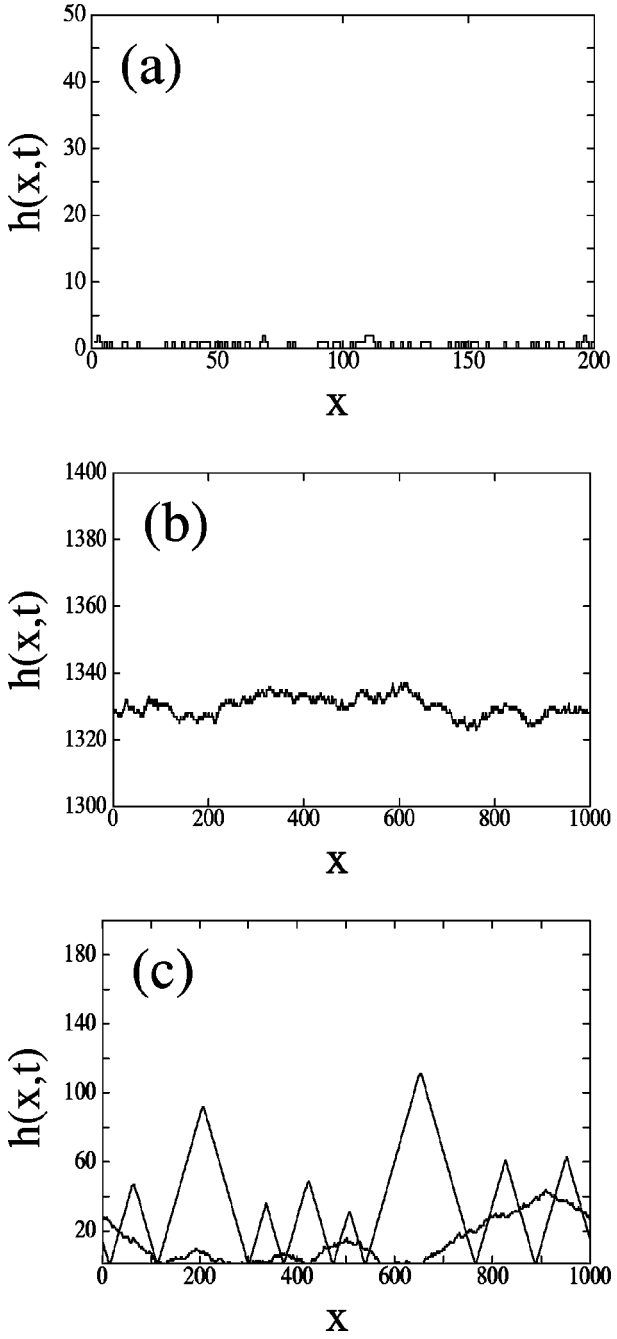

FIG. 8. Snapshots of the interface for $p=0.4$ (a), 0.5 (b), and 0.8 (c). In this figure, the system sizes are $L=200$ in (a) and $L=1000$ in (b) and (c).

Consequently, our model shows various morphological changes as p increases from 0 to 1. For $p<p_{c1}$, the interface is very smooth. But the interface becomes rough at p_{c1}. The interface becomes rougher than at p_{c1} as p increases from p_{c1} to p_{c2}. For $p>p_{c2}$, the interface has a shape like a line of pyramids. We draw typical interface configurations for various p in Fig. 8.

Recently, two growth models, which are related to the PC universality class at the reference height, were introduced. One is the model introduced by Park and Kahng (PK) [3]. The other is the model introduced by Hinrichsen and Ódor (HÓ) [2]. In the PK model, the dynamics of the particles at the bottom layer has Z_2-symmetric absorbing states if any deposition of a particle above the bottom layer is not allowed. When the deposition of a particle above the bottom layer is allowed, however, their model shows very different dynamics from the PC dynamics even at the bottom layer. On the other hand, the HÓ model, where the dynamics of the particles at the bottom layer also follows the PC dynamics, shows the same dynamics as that of the PC dynamics when the deposition of a particle above the bottom layer is allowed. Park and Kahng argued that the unexpected behavior may be related to the fact that the dynamical processes at lower levels are strongly suppressed by the particles at higher levels. In particular, kinks between different species of particles may become frozen when they are covered with another layer of particles. The growth rule of our model seems to be similar to that of the PK model, but there exists no frozen effect blocking the dynamics of the interface at the bottom layer. In our model, a particle cannot be deposited at a site if at least one different kind of a particle exists at its nearest-neighbor sites after deposition. In the PK model, however, two different kinds of particles or two same kinds of particles can locate conditionally at the nearest-neighbor sites of each other. This rule leads to the suppression effect in the PK model and a very strange dynamical behavior that is different from the PC dynamics. In the HÓ model, the PC dynamics is satisfied at each layer of the growing interface. Therefore, in this model there is no strange behavior such as the suppression effect.

In summary, we have introduced a simple growth model exhibiting two first-order BU transitions. The first-order BU transition at p_{c1} occurs because of difference in the growing dynamics between the bottom layer and other layers except the bottom layer. In the bottom layer, the growth of the interface in our model is related to the PC-type dynamics. In other layers except the bottom layer, however, the growth dynamics is related to the DP-type dynamics. We measured the roughness exponent ζ at the BU transition point p_{c1}. We found that the value of ζ is 0.43(2). The value of the roughness exponent jumps from 0 to 0.43 at p_{c1} discontinuously because of $\zeta=0$ for $p<p_{c1}$. Therefore, our model shows a nonequilibrium roughening transition at p_{c1}. We found that our model also shows another first-order BU transition at p_{c2}. Our model shows a smoothing transition at p_r when p decreases from $p(>p_r)$ to 0 continuously after the width of the interface reaches a constant value.

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